SOLUTIONS:

1)

$$\boxed{e^{ix} = 1 + \frac{(ix)}{1!} + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \frac{(ix)^7}{7!} + \cdots}$$

2)

$$\boxed{e^{ix} = 1 + \frac{ix}{1!} + \frac{-x^2}{2!} + \frac{-ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} + \frac{-x^6}{6!} + \frac{-ix^7}{7!} + \cdots}$$

3)

$$\boxed{e^{ix} = \left(1 + \frac{-x^2}{2!} + \frac{x^4}{4!} + \frac{-x^6}{6!} + \cdots\right) + i\left(\frac{x}{1!} + \frac{-x^3}{3!} + \frac{x^5}{5!} + \frac{-x^7}{7!} + \cdots\right)}$$

4)

$$e^{a+bi} = e^a e^{bi}$$

$$\boxed{e^{a+bi} = e^a \cos b + i\, e^a \sin b}$$

5)

$$\boxed{e^{-ix} = \cos x - i \sin x}$$

6)

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

$$e^{ix} + e^{-ix} = 2\cos x$$

$$\boxed{\cos x = \frac{e^{ix} + e^{-ix}}{2}}$$

$$e^{ix} - e^{-ix} = 2i \sin x$$

$$\frac{e^{ix} - e^{-ix}}{2i} = \sin x$$

$$-i\frac{e^{ix} - e^{-ix}}{2} = \sin x$$

$$\boxed{\sin x = i\frac{e^{-ix} - e^{ix}}{2}}$$

7)

$$\sin ix = i\frac{e^{-i(ix)} - e^{i(ix)}}{2}$$

$$\boxed{\sin ix = i\frac{e^{x} - e^{-x}}{2}}$$

$$\cos ix = \frac{e^{i(ix)} + e^{-i(ix)}}{2}$$

$$\boxed{\cos ix = \frac{e^{x} + e^{-x}}{2}}$$

8)

$$\cot ix = \frac{1}{\tan ix}$$

$$\cot ix = \frac{1}{i\tanh x}$$

$$\cot ix = \frac{1}{i} \cdot \frac{1}{\tanh x}$$

$$\boxed{\cot ix = -i\coth x}$$

9)

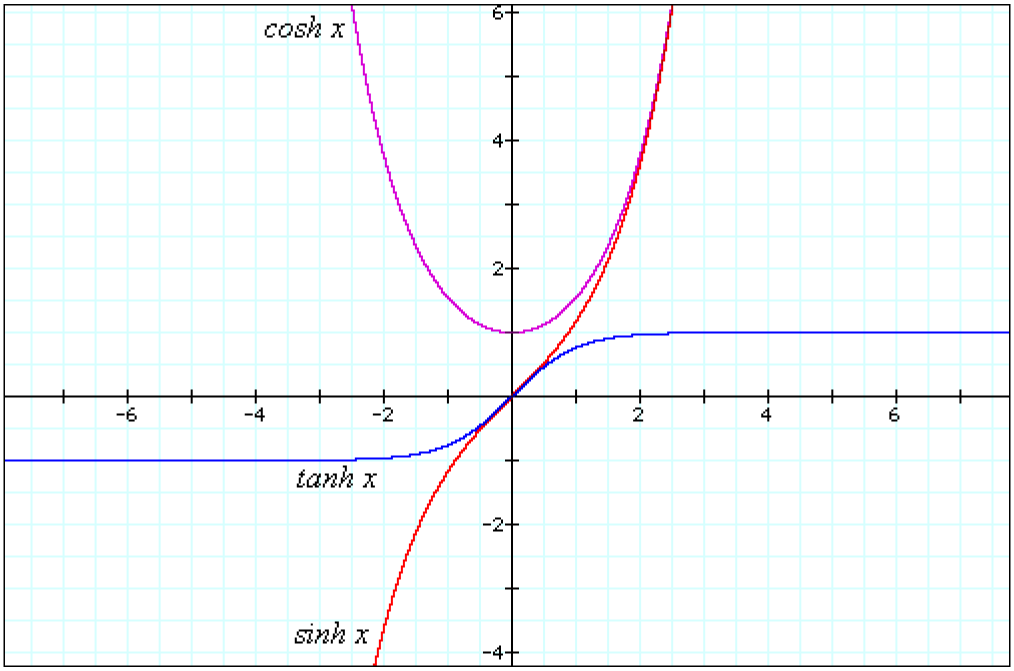
$$\sin(a + bi) = \sin a \cos bi + \cos a \sin bi$$

$$\boxed{\sin(a + bi) = \sin a \cosh b + i\cos a \sinh b}$$

$$\cos(a + bi) = \cos a \cos bi - \sin a \sin bi$$

$$\boxed{\cos(a + bi) = \cos a \cosh b - \sin a \sinh b}$$

10)



11)

$$a = \frac{\pi}{2} + n\pi \quad \text{or} \quad b = 0$$

12)

When b = 0, the real term cannot be set equal to 2. Therefore

When $a = \frac{3\pi}{2} + n2\pi$, $\sin a = -1$. This cannot be a solution since $\cosh x \geq 1$.

Therefore, $a = \frac{\pi}{2} + n2\pi$, $\sin a = 1$, and $b = \pm\cosh^{-1} 2$

$$\cosh b = 2$$
$$\frac{e^b + e^{-b}}{2} = 2$$
$$e^b + e^{-b} = 4$$
$$e^{2b} + 1 = 4e^b$$
$$\left(e^b\right)^2 - 4e^b + 1 = 0$$
$$e^b = \frac{4 \pm \sqrt{16 - 4\cdot1\cdot1}}{2}$$
$$e^b = 2 \pm \sqrt{3}$$
$$\boxed{b = \ln\left(2 \pm \sqrt{3}\right)}$$

13)

Since $2 + \sqrt{3} = \frac{1}{2 - \sqrt{3}}$, we can say $b = \pm\ln\left(2 + \sqrt{3}\right)$.

$$\sin^{-1} 2 = \frac{\pi}{2} + n2\pi \pm i\ln\left(2 + \sqrt{3}\right)$$

The principal value is when $n = 0$ and the imaginary term is positive.

$$\boxed{\begin{aligned}&\sin^{-1} 2 = \frac{\pi}{2} + i\ln\left(2 + \sqrt{3}\right)\\ &\sin^{-1} 2 \approx 1.57080 + 1.31696\, i\end{aligned}}$$

14)

$$\left(r_1e^{i\theta_1}\right)\left(r_2e^{i\theta_2}\right) = r_1r_2e^{i\theta_1}e^{i\theta_2}$$
$$\boxed{\left(r_1e^{i\theta_1}\right)\left(r_2e^{i\theta_2}\right) = \left(r_1r_2\right)e^{i(\theta_1+\theta_2)}}$$

15)

$$\boxed{\left(re^{i\theta}\right)^n = \left(r^n\right)e^{i\theta n}}$$

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16)

$$\left(2e^{i\pi/3}\right)^{100} = 2^{100} e^{i100\pi/3}$$
$$\left(2e^{i\pi/3}\right)^{100} = 2^{100} e^{i(33+1/3)\pi}$$

Remember that in polar coordinates $\theta \equiv \theta + n2\pi$. This means that $e^{i(33+1/3)\pi}$ can be reduced by 32π.

$$\left(2e^{i\pi/3}\right)^{100} = 2^{100} e^{i\frac{4\pi}{3}}$$

This can be converted back to rectangular coordinates.

$$\left(2e^{i\pi/3}\right)^{100} = 2^{100}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$$
$$\left(2e^{i\pi/3}\right)^{100} = 2^{100}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$
$$\boxed{\left(2e^{i\pi/3}\right)^{100} = -2^{50} - i\,2^{50}\sqrt{3}}$$

17)

For multiplication, multiply the lengths and add the angles.

For exponentiation, raise the length to the power and multiply the angle by the power.

18)

$$\sqrt[n]{re^{i\theta}} = \sqrt[n]{r}\sqrt[n]{e^{i\theta}}$$
$$\sqrt[n]{re^{i\theta}} = \sqrt[n]{r}\,e^{i\theta/n}$$

There are $n-1$ more values that occur when θ is increased by multiples of 2π.

$$\boxed{\sqrt[n]{re^{i\theta}} = \sqrt[n]{r}\,e^{i(\theta+k2\pi)/n},\ k = 0, 1, \ldots, n-1}$$

19)

$$i = e^{i\frac{\pi}{2}}$$
$$i^i = \left(e^{i\frac{\pi}{2}}\right)^i$$
$$i^i = e^{ii\frac{\pi}{2}}$$
$$\boxed{i^i = e^{-\pi/2} \approx 0.20788}$$

20)

$$i = e^{i(\pi/2 + n2\pi)}$$

$$i^i = e^{-(\pi/2 + n2\pi)}$$

$$\boxed{i^i = e^{-\pi/2 + n2\pi}}$$